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Paschen–Back Effect in Dyonium

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A formulation of the Pashen–Back effect in dyonium is discussed to explain the recent evidence for a magnetic monopole of mass 2397 MeV and Dirac charge $g=(137/2)e$. The masses for isospin $I=0$ mesons are estimated and compared with experiment.

In this paper I discuss the existence of the Paschen–Back effect in dyonium (Barut, 1971) and offer an explanation for the recent evidence of a magnetic monopole (Akers, 1986).

Recent evidence suggests that a low-mass magnetic monopole of Dirac charge $g=(137/2)e$ interacts with a c-quark's magnetic dipole moment to produce Zeeman splitting of meson states. I now suggest that the interaction is between a pair of dyons with electric and magnetic charges (Ze, g) and $(-Ze, -g)$, where Z may be integral or fractional (Schwinger, 1969).

Barut has studied extensively the bound states for dyonium through solution of the Schrödinger, Klein-Gordon, and Dirac equations (Barut and Bornzin, 1971). Barut finds the energy eigenvalues to the bound states, and they form an overcomplete set in the case of a large attractive potential associated with a magnetic charge. The binding energy of the s state for dyonium is (Frauenfelder and Henley, 1974)

$$E=-\alpha_s^2 M_0 c^2/4=-g^2/2a_0 \tag{1}$$

where a_0 is the "Bohr" radius of the system. For the dyon with magnetic charge $g=(137/2)e$, $\alpha_s=34.25$ and $M_0c^2=2397$ MeV (Akers, 1986). This gives a binding energy $E=-703$ GeV for the ground state. With an ionization energy of 703 GeV required for dyonium to be separated, we can understand why recent accelerator searches have been unsuccessful up to

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the 540-GeV center-of-mass energy. Of course, there still may be a confinement mechanism for magnetic monopoles to prevent their appearance.

The referee of Akers (1986) suggested solving the Schröndinger equation for concrete energy solutions of the system (Ze, g) and $(-Ze, -g)$. It is possible to consider the free-particle bound states for dyonium from the Schrödinger and Dirac equations (Barut and Bornzin, 1971). The free-particle bound states of isospin $I=0$ mesons have the energy eigenvalues:

$$E_{nm_Lm_I} = M_0c^2 + nm_Ig_ME^Z_{nm_Lm_I} + nm_Lm_Ig_ME^{SO}_{nm_Lm_I} + nm_Ig_ME^{hf}_{nm_Lm_I}\left[\frac{8\pi}{3}|\Psi_{nL}(0)|^2\langle\mathbf{I}\cdot\mathbf{S}\rangle + \left\langle\frac{3(\mathbf{I}\cdot\mathbf{R})(\mathbf{S}\cdot\mathbf{R}) - \mathbf{I}\cdot\mathbf{S}}{R^3}\right\rangle\right] \tag{2}$$

where $\mathbf{I}$ and $\mathbf{S}$ are the intrinsic spins of the dyons with angular momentum $\frac{1}{2}$ each (Osborn, 1982; Akers and Akers, 1984). The intrinsic spin $\mathbf{I}$ is not to be confused with the isopin I of the mesons. The first term in equation (2) comes from the rest mass in the Dirac equation. The second term is the Zeeman effect due to the electric and magnetic dipole moments interacting with their respective fields:

$$nm_Ig_ME^Z_{nm_Lm_I} = \boldsymbol{\mu}\cdot\mathbf{B} + \mathbf{m}\cdot\mathbf{E} \tag{3}$$

where $\boldsymbol{\mu}$ is the magnetic dipole moment and $\mathbf{m}$ is the electric dipole moment. The third and fourth terms in equation (2) are, respectively, the spin-orbit and hyperfine energy splittings. The Landé g-factor is

$$g_M = 1 + \frac{J(J+1) + I(I+1) - L(L+1)}{2J(J+1)} \tag{4}$$

For spin singlet states, $L=0$ and $g_M=2$; for spin triplet states, $L=1$ and $g_M = 4/3$. The wave function in equation (2) is the hydrogen-type function with the "Bohr" radius a_0 of the system (Ze, g) and $(-Ze, -g)$. In addition, $\langle\mathbf{I}\cdot\mathbf{S}\rangle = 1$ for spin triplet states and -3 for spin singlet states.

In a strong field associated with a magnetic charge g, the level splitting of the Zeeman and spin-orbit terms is called the Paschen-Back effect (Baym, 1969). The good quantum numbers are then m_L and m_I in a strong magnetic field (Anderson, 1971). $m_I = -\frac{1}{2}, \frac{1}{2}$ for $I=\frac{1}{2}$; $m_L = -L, \ldots, 0, \ldots, L$.

The masses of isospin $I=0$ mesons are calculated from equation (2) by fitting the coefficients $E^Z_{nm_Lm_I}$, $E^{SO}_{nm_Lm_I}$, and $E^{hf}_{nm_LM_I}$ to the center-of-mass splittings in the spectrum of charmonium. For the $n=1$ shell structure of equation (2), the coefficients are

$$E^{Z}_{1m_Lm_I} = 670 \text{ MeV}, \qquad E^{SO}_{11m_I} = 926 \text{ MeV}$$
$$E^{hf}_{10m_I} = 43.5 \text{ MeV}, \qquad E^{hf}_{11m_I} = 70 \text{ MeV}$$

For example, to calculate the mass of the $\Psi(3097)$ meson from equation (2), the quantum numbers are $m_L = 0$ and $m_I = \frac{1}{2}$ for this spin triplet state. Now, $\langle \mathbf{I} \cdot \mathbf{S} \rangle = 1$ and $g_M = 2$. Thus,

$$E = M_0c^2 + m_I g_M E^{Z}_{10m_I} + m_I g_M E^{hf}_{10m_I} \cdot \frac{8}{3}\pi |\Psi_{10}(0)|^2 \langle \mathbf{I} \cdot \mathbf{S} \rangle$$
$$= 2397 + \tfrac{1}{2}(2)670 + \tfrac{1}{2}(2)43.5\left(\frac{8}{3}\pi\right)\left(\frac{1}{4\pi}\right) \cdot 1 = 3096 \text{ MeV}$$

The masses for isospin $I = 0$ mesons are calculated from equation (2) and are given in Table I. In Table I, experimental masses are shown in parentheses next to the calculated values. The experimental masses are from the Particle Data Group (1986). In Table I, mesons with resonances not established are shown in brackets. The theoretical mesons without associated experimental masses in Table I may be suppressed (forbidden) due to parity conservation. The meson in the $J^{PC} = 0^{-+}$ bin is predicted to exist (Akers, 1986). This is shown in Figure 1, where mesons with experimental masses are shown. The dotted lines are the theoretical values, and the solid lines are the experimental masses. The mesons ε, D, and f have the new names f_0, f_1, and f_2, respectively (Particle Data Group, 1986). The calculated values for these mesons are within experimental uncertainties. For $\varepsilon(1300)$ the experimental uncertainty is shown in Figure 1.

One can envisage several improvements of our present discussion. The coefficients for $n = 2$ in equation (2) may be fitted with the 2^1S_0 and 2^3S_1 states of charmonium once experimental uncertainties are reduced.

Table I. Energies of Free-Particle Bound States for $I = 0$ Mesons[a]

Quantum numbers		Energy, MeV				
		1^1S_0	1^3S_1	1^3P_0	1^3P_1	1^3P_2
J^{PC}		0^{-+}	1^{--}	0^{++}	1^{++}	2^{++}
$m_L = 1$,	$m_I = \frac{1}{2}$			3415 (3415)	3505 (3510)	3551 (3555)
$m_L = 0$,	$m_I = \frac{1}{2}$	2980 (2980)	3096 (3097)	2798	2890	2936
$m_L = -1$,	$m_I = \frac{1}{2}$			2613	2521	2475
$m_L = -1$,	$m_I = -\frac{1}{2}$			2181	2273	2318 [2240]
$m_L = 0$,	$m_I = -\frac{1}{2}$	1814	1698 (1680)	1996	1904	1858 [1810]
$m_L = 1$,	$m_I = -\frac{1}{2}$			1379 (1300)	1287 (1285)	1241 (1270)

[a]The usual spectroscopic notation is shown, along with established experimental masses in parentheses and nonestablished resonances in brackets.

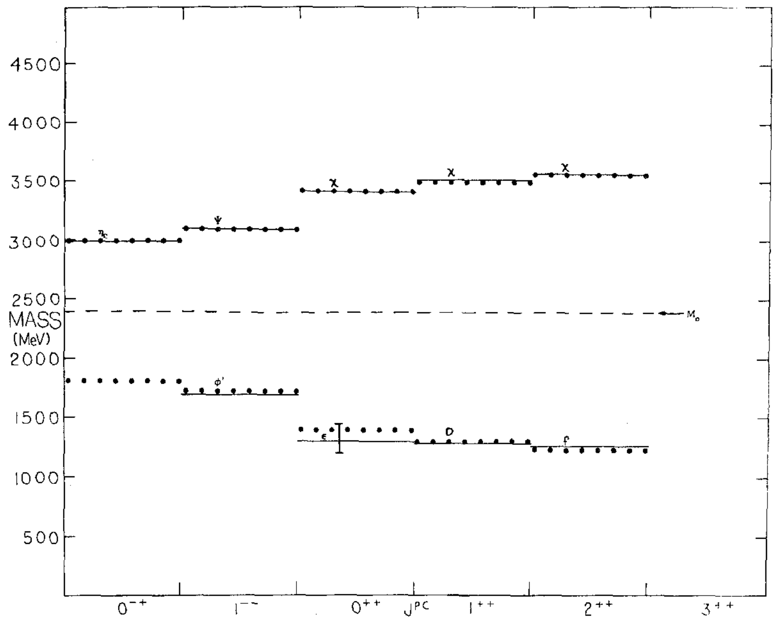

Fig. 1. Theoretical values of isospin $I=0$ mesons are indicated by dotted lines and experimental masses by solid lines. $M_0c^2 = 2397$ MeV is the mass of the dyon in this paper. A missing η meson is indicated in the $J^{PC} = 0^{-+}$ bin. Its mass is estimated to be 1814 MeV.

Extension of the Paschen–Back effect to nonisoscalar mesons should be studied. Barut (1971) has developed a model of the proton based upon dyons. Perhaps his model can be extended to other baryons as well. A theory of the leptons has also been developed by Barut (1979). It may be worth investigating these ideas further.

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